Suppose there are three missionaries and three cannibals who need to cross to the far side of a river using a single boat that can carry one or two people at a time. Both groups will cooperate and can paddle back and forth freely, but old habits will lead the cannibals to eat the missionaries if the missionaries are ever outnumbered on either side of the river.

The problem is to find a way to get all of the missionaries and all of the cannibals safely across the river.

You may write it in a different way than what is suggested below, but it must implement a depth-first or breadth-first search to find the answer.

General tips

Carefully test each word you write before moving on. The design suggested here makes it straightforward to test most words in isolation, but some require a bit more work to test.

If a word does not work, carefully track its use of the stack. Break each and every word onto its own line, and document the state of the stack after that line. For example, you could rewrite this word:

```
\ check if a number is a member of the used set
: isused ( n -- bool )
  \ loop through all set elements
  usedcounter @ 0 ?do
    \ compare n with elt i
    dup used i cells + @
    \ return true if its a match
    = if drop -1 unloop exit then
  loop
  \ return false
  drop 0
;
```

to be like this:

```
\ check if a number is a member of the used set
: isused ( n -- bool )
  \loop through all set elements
  usedcounter @ ( n &usedcounter )
  @ ( n usedcounter )
  0 ( n usedcounter 0 )
  ?do ( n )
    \ compare n with elt i
    dup ( n n )
    used ( n n &used )
    i ( n n &used i )
    cells ( n n &used i*8 )
    + ( n n &used[i] )
    @ ( n n used[i] )
    \ return true if its a match
    = ( n n==used[i] )
    if ( n )
      drop ( )
      -1 ( true )
      unloop ( true )
      exit ( true )
    then ( n )
    loop ( n )
  \ return false
  drop ( )
  0 ( false )
;```
If you are meticulous about documenting how the stack is used, it is much easier to catch low-level mistakes. It can make it harder to read and follow the higher-level flow of the word, however, so use this technique judiciously. For example, you might write a word this way, test it, and then convert it into the first version (shown above) once you are confident that it is correct.

**Overview**

I suggest representing states in two ways:

- Store three values on the stack for a single state: `near`, `m`, and `c`. `near` is true if the boat is on the near side of the river. `m` and `c` are the number of missionaries and cannibals on the near side of the river, respectively.

- Store all three values in a single integer. This is the “packed” format. While less convenient to work with, this format makes it easier to store states in stacks and sets.

The overall flow of the program will follow this pseudo-code:

- push the start state on the candidate stack
- search:
  - print the candidate stack
  - pop a candidate state off the candidate stack
  - push a copy on the bread-crumbs trail stack
  - if it is the goal state
    - print out the contents of the bread-crumbs trail in order. this is the solution to the puzzle.
  - else
    - generate a list of successor states (there should be exactly 5)
    - push the valid, legal, fresh successors on the candidate stack
    - for each successor generated in this step:
      - call search recursively
    - pop the state off the bread-crumbs trail stack

I suggest writing lots of helper words. Avoid complexity in the search word, since it will already be the most difficult part to test. What follows is one suggested implementation. You are free to write it a different way if you would prefer.

**Implementation**

Start by writing some helper words. The first few are described adequately by their names and stack-effect comments:

\[
\begin{align*}
: 3dup & \ ( x \ y \ z \ -- \ x \ y \ z \ x \ y \ z ) \ ... \ ; \\
: 3drop & \ ( x \ y \ z \ -- ) \ ... \ ; \\
: pack & \ ( \textit{near} \ m \ c \ -- \ packedstate ) \ ... \ ; \\
: unpack & \ ( \textit{packedstate} \ -- \ near \ m \ c ) \ ... \ ; \\
: printstate & \ ( \textit{side} \ m \ c \ -- ) \ ... \ ; \\
\end{align*}
\]

Write each of these and test them thoroughly. To load code from a source file, give the name of the source file when loading gforth:

```
gforth missionaries.fs
```

Next, implement basic stack and set data structures:

\[
\begin{align*}
\\text{test if } n \ \text{is in the used set} & \; \\
: \text{isused } & \ ( n \ -- \ bool ) \ ... \ ; \\
\\text{add } n \ \text{to the used set} & \; \\
: \text{addused } & \ ( n \ -- ) \ ... \ ; \\
\\text{push a value on the candidate stack} & \; \\
: \text{pushcandidate } & \ ( n \ -- ) \ ... \ ; \\
\\text{pop a value off the candidate stack} & \; \\
: \text{popcandidate } & \ ( -- n ) \ ... \ ; \\
\end{align*}
\]
\push a value on the bread crumb trail stack
: pushcrumb ( n -- ) ... ;

\pop a value off the bread crumb trail stack
: popcrumb ( -- n ) ... ;

For debugging, I suggest writing words to print out the entire contents of each of these data structures:

\print the contents of the used set in order
: printused ( -- ) ... ;

\print the contents of the candidate stack in order
: printcandidates ( -- ) ... ;

\print the contents of the bread crumb trail in order
: printcrumbs ( -- ) ... ;

Next, start the words to work with states:

\push the starting state onto the stack
: startstate ( -- near m c ) ... ;

\test if the state on the stack is the goal state
: isgoal ( near m c -- bool ) ... ;

\test if the state on the stack is valid and legal
: isvalid ( near m c -- bool ) ... ;

The next two generate, record, and report on potential next moves from a given state:

\add a state to the candidate stack if it is valid and new
\report on the outcome: invalid, repeat, or fresh
: addcandidate ( near m c -- ) ... ;

\find all successor candidates for the given state, push them on stack
\leaves the number of states added on the stack
: successors ( near m c -- n )

\addcandidate requires a bit more explanation. It should:

- check if the (packed) state is valid (reject it and print a message if not)
- check if the (packed) state is already used (reject it and print a message if not)
- otherwise:
  - print a message indicating that it is a fresh state
  - add the packed state to the used list
  - add the packed state to the candidate stack

\addcandidate is a helper for \successors, which generates all (5) possible successors for a given state, handing each one to \addcandidate, which filters some of them out and adds the rest to the candidate stack.

Then comes the main search loop:

\find the solution from position at top of stack
: search ( -- )

\search assumes that a state is already on the candidate stack, and it uses the words already described to implement the pseudo-code given earlier. For a word to call itself recursively, it uses the special word \recurse instead of the normal word name (\search in this case).

Finally, write a \start word that resets the stacks and the used set, puts the start state on the candidate stack, adds it to the used set, and calls \search.

\textbf{Example}

Here is the output of my solution:

$ gforth missionaries.fs
start
candidates:
[ near 3 3 ]
fresh [ far 3 2 ]
fresh [ far 3 1 ]
fresh [ far 2 2 ]
invalid [ far 2 3 ]
invalid [ far 1 3 ]
candidates:
[ far 3 2 ]
[ far 3 1 ]
[ far 2 2 ]
invalid [ near 2 3 ]
invalid [ near 2 4 ]
repeat [ near 3 3 ]
fresh [ near 3 2 ]
invalid [ near 4 2 ]
candidates:
[ far 3 2 ]
[ far 3 1 ]
[ far 2 2 ]
repeat [ far 3 1 ]
fresh [ far 3 0 ]
invalid [ far 2 1 ]
repeat [ far 2 2 ]
invalid [ far 1 2 ]
candidates:
[ far 3 2 ]
[ far 3 1 ]
[ far 3 0 ]
fresh [ near 3 1 ]
repeat [ near 3 1 ]
invalid [ near 4 1 ]
invalid [ near 4 0 ]
invalid [ near 5 0 ]
candidates:
[ far 3 2 ]
[ far 3 1 ]
[ near 3 1 ]
repeat [ far 3 0 ]
invalid [ far 3 -1 ]
invalid [ far 2 0 ]
invalid [ far 2 1 ]
fresh [ far 1 1 ]
candidates:
[ far 3 2 ]
[ far 3 1 ]
[ far 1 1 ]
invalid [ near 1 2 ]
invalid [ near 1 3 ]
fresh [ near 2 2 ]
invalid [ near 2 1 ]
repeat [ near 3 1 ]
candidates:
[ far 3 2 ]
[ far 3 1 ]
[ near 2 2 ]
invalid [ far 2 1 ]
invalid [ far 2 0 ]
repeat [ far 1 1 ]
invalid [ far 1 2 ]
fresh [ far 0 2 ]
candidates:
candidates:
[ far 3 2 ]
[ far 3 1 ]
[ far 0 2 ]
fresh [ near 0 3 ]
invalid [ near 0 4 ]
invalid [ near 1 3 ]
invalid [ near 1 2 ]
repeat [ near 2 2 ]

candidates:
[ far 3 2 ]
[ far 3 1 ]
[ near 0 3 ]
repeat [ far 0 2 ]
fresh [ far 0 1 ]
invalid [ far 0 0 ]
invalid [ near 1 2 ]
fresh [ near 1 1 ]
invalid [ near 2 1 ]

candidates:
[ far 3 2 ]
[ far 3 1 ]
[ far 0 1 ]
fresh [ near 0 2 ]
repeat [ near 0 3 ]
invalid [ near 0 3 ]
fresh [ near 1 1 ]
invalid [ near 2 1 ]

candidates:
[ far 3 2 ]
[ far 3 1 ]
[ far 0 2 ]
[ near 1 1 ]
invalid [ far 1 0 ]
invalid [ far 1 -1 ]
fresh [ far 0 0 ]
repeat [ far 0 1 ]
invalid [ far -1 1 ]

solution found
-------------------
[ near 3 3 ]
[ far 2 3 ]
[ near 3 2 ]
[ far 3 0 ]
[ near 3 1 ]
[ far 1 1 ]
[ near 2 2 ]
[ far 0 2 ]
[ near 0 3 ]
[ far 0 1 ]
[ near 1 1 ]
[ far 0 0 ]
backtracking
backtracking

candidates:
[ far 3 2 ]
[ far 3 1 ]
[ near 0 2 ]
repeat [ far 0 1 ]
repeat [ far 0 0 ]
invalid [ far -1 1 ]
invalid [ far -1 2 ]
invalid [ far -2 2 ]
backtracking
backtracking
backtracking
backtracking
backtracking
backtracking
backtracking
backtracking
backtracking
backtracking
backtracking

candidates:
[  far 3 2 ]
[  far 3 1 ]
repeat [ near 3 2 ]
repeat [ near 3 3 ]
invalid [ near 4 2 ]
invalid [ near 4 1 ]
invalid [ near 5 1 ]
backtracking

candidates:
[  far 3 2 ]
repeat [ near 3 3 ]
invalid [ near 3 4 ]
invalid [ near 4 3 ]
invalid [ near 4 2 ]
invalid [ near 5 2 ]
backtracking
backtracking
ok